

ARTIFICIAL VARIABLES TECHNIQUE

Introduction

LPP in which constraints may also have $>$, and $=$ signs after ensuring that all $b_i \geq 0$ are considered in this section. In such cases basis matrix cannot be obtained as an identity matrix in the starting simplex table, therefore we introduce a new type of variable called the *artificial variable*. These variables are fictitious and cannot have any physical meaning. The artificial variable technique is a device to get the starting basic feasible solution, so that simplex procedure may be adopted as usual until the optimal solution is obtained. To solve such LPP there are two methods.

- (i) The Big M Method or Method of Penalties.
- (ii) The Two-phase Simplex Method.

The Big M Method

The following steps are involved in solving an LPP using the Big M method.

Step 1 Express the problem in the standard form.

Step 2 Add non-negative artificial variables to the left side of each of the equations corresponding to constraints of the type $>$, or $=$. However, addition of these artificial variable causes violation of the corresponding constraints. Therefore, we would like to get rid of these variables and would not allow them to appear in the final solution. This is achieved by assigning a very large penalty ($-M$ for maximization and $+M$ for minimization) in the objective function.

Step 3 Solve the modified LPP by simplex method, until any one of the three cases may arise.

1. If no artificial variable appears in the basis and the optimality conditions are satisfied, then the current solution is an optimal basic feasible solution.
2. If at least one artificial variable in the basis at zero level and the optimality condition is satisfied then the current solution is an optimal basic feasible solution (though degenerated solution).
3. If at least one artificial variable appears in the basis at positive level and the optimality condition is satisfied, then the original problem has no feasible solution. The solution satisfies the constraints but does not optimise the objective function, since it contains a very large penalty M and is called *pseudo optimal solution*.

Note: While applying simplex method, whenever an artificial variable happens to leave the basis, we drop that artificial variable and omit all the entries corresponding to its column from the simplex table.

Example 1 Use penalty method to Maximize $Z = 3x_1 + 2x_2$ Subject to the constraints

$$\begin{aligned}2x_1 + x_2 &\leq 2 \\3x_1 + 4x_2 &\geq 12 \\x_1, x_2 &\geq 0\end{aligned}$$