

Question 1

The following table gives the activities of a construction project and other relevant information.

- (a) What is the normal project length and the minimum project length?
 (b) Determine the minimum crashing costs of schedule ranging from normal length down to and including the minimum length schedule.

Activity i- j	Normal duration (days)	Crash duration (days)	Cost of crashing (Rs per day)
1-2	9	6	20
1-3	8	5	25
1-4	15	10	30
2-4	5	3	10
3-4	10	6	15
4-5	2	1	40

- (c) What is the optimal length schedule duration of each job for your solution?
 Overhead of the project is Rs. 60 per day.

Question 2

The table below provides cost and estimates for a seven activity project.

Activity	Time estimate (weeks)		Direct cost estimate (Rs. 1000)	
	Normal	Crash	Normal	Crash
A 1-2	2	1	10	15
B 1-3	8	5	15	21
C 2-4	4	3	20	24
D 3-4	1	1	7	7
E 3-5	2	1	8	15
F 4-6	5	3	10	16
G 5-6	6	2	12	36

- (a) Draw the project network corresponding to normal time.
 (b) Determine the critical path and the normal duration and cost of the project.
 (c) Crash the activities so that the project completion time reduces to 9 weeks.

Question 3

A Company manufactures purses, shaving bags, and backpacks. The construction of the three products requires lather and synthetic material, with leather being the limiting raw material. The production process uses two types of skilled labor: sewing and finishing. The following table gives the availability of the resources, their usage by the three products, and the profits per unit.

Resource requirements per unit				
Resource	Purse	Bag	Backpack	Daily availability
Leather (ft^2)	2	1	3	42

Sewing(hr)	2	1	2	40
Finishing (hr)	1	5	1	45
Profit (\$)	24	22	45	

Formulate the problem as a linear program and find the optimum solution. Next, indicate whether the following changes in the resources will keep the current solution feasible. For the cases where feasibility is minted, determine the new optimum solution (values of the variables and the objective function).

- (a) Available leather is increased to 45 ft².
- (b) Available leather is decreased by 1ft².
- (c) Available sewing hours are changed to 38 hours.
- (d) Available sewing hours are changed to 46 hours.
- (e) Available finishing hours are decreased to 15 hours.
- (f) Available finishing ours are increased to 50 hours.
- (g) Would you recommend hiring an additional sewing worker at \$ 15 an hour?

Question 4

A company produces two models of electronic gadgets that resistors, capacitors, and chips. The following table summarizes the data of the situation:

Resource	Unit resource		Maximum availability (units)
	Model 1 (units)	Model 2 (units)	
Resistor	2	3	1200
Capacitor	2	1	1000
Chips	0	4	800
Unit profit (\$)	3	4	

Let x_1 and x_2 be the amounts produced of Models 1 and 2, respectively. Following are the LP model and its associated optimal simplex tableau.

$$\text{Maximize } z = 3x_1 + 4x_2$$

Subject to

$$2x_1 + 3x_2 \leq 1200 \quad (\text{Resistors})$$

$$2x_1 + x_2 \leq 1000 \quad (\text{Capacitors})$$

$$4x_2 \leq 800 \quad (\text{Chips})$$

$$x_1, x_2 \geq 0$$

Basic	X_1	X_2	S_1	S_2	S_3	Solution
Z	0	0	5/4	1/4	0	1750
X_1	1	0	-1/4	3/4	0	450
S_3	0	0	-2	2	1	400
X_2	0	1	1/2	-1/2	0	100

- (a) Determine the status of each resources.
- (b) In terms of the optimal profit, determine the worth of resistor. One capacitor One chip.
- (c) Determine the range of applicability of the dual prices for each resource.
- (d) If the available number of resistors is increased to 1300units, find new optimum
- (e) If the available number of chips is reduced to 350 units, will you ale to determine the new optimum solution directly from the given information? Explain.

- (f) A new contractor is offering to sell additional resistors at 40 cents each but only if the company would purchase at least 500 units. Should you advice to accept the offer?

Question 5

A particular company has a daily budget of 320 hours of labour and 350 units of raw material to manufacture two products. If necessary, the company can employ up to 10 hours daily of overtime labour hours at the additional cost of \$ 2 an hour. It takes 1 labour hour and 3 units of raw material to produce one unit of product 1 , and 2 labour hours and 1 unit of raw material to produce 1 unit of product 2 . The profit per unit of product 1 is \$ 10 , and that of product 2 is \$ 12 , Let x_1 and x_2 define the daily number of unit produced of products 1 and 2 ,, and x_3 the daily hours of overtime used. The LP model and its associated optimal simplex tableaue are then given as

$$\text{Maximize } z = 10x_1 + 12x_2 - 2x_3$$

Subject to

$$x_1 + 2x_2 - x_3 \leq 320 \quad (\text{Labour hours})$$

$$3x_1 + x_2 \leq 350 \quad (\text{Raw material})$$

$$x_3 \leq 10 \quad (\text{Overtime})$$

$$x_1, x_2, x_3 \geq 0$$

Basic	X_1	X_2	X_3	S_1	S_2	S_3	Solution
z	0	0	2	5.2	1.6	0	2224
X_2	0	1	0	0.6	-0.2	0	122
X_1	1	0	0	-0.2	0.4	0	76
S_3	0	0	1	0	0	1	10

- Determine the optimal solution of the problem.
- Determine the dual prices and the applicability ranges of their associated resources.
- Examine the dual prices for labour hours (constraint 1) and overtime hours (constraint 3). Shouldn't these two values be the same? Explain.
- If the company can acquire an additional 100 units of raw material daily at \$ 1.50 a unit, would you advice the company to do so? What is the cost of raw material is \$ 2 a unit.
- Suppose that the company can acquire at most 200 additional units of raw material a day, determine the associated optimal solution.
- Suppose that the company can use no more than 8 hours of overtime daily; find the new optimum the solution.

Question 6

The following table gives the cost of transporting material from supply points A, B, C and D to demand points E, F, G, H and J.

	E	F	G	H	J	Supply
A	8	10	12	17	15	100
B	15	13	18	11	9	150
C	14	20	6	10	13	180
D	13	19	7	6	12	280
Demand	90	170	50	210	190	

The present allocation is as follows:

A to E 90, A to F 10, B to F 150, C to F 10, C to G 50, C to J 120, D to

H 210, D to J 70.

- (a) Check the allocation is optimum, if not find an optimum schedule.
(b) If in the above problem the transportation cost from A to G is reduced to 10, what will be the new optimum schedule?
(c) If the availability of supply point A is reduced by 10 units, use each of the following criteria to obtain a initial basic feasible solution:
(i) Northwest corner rule
(ii) Least cost method
- (d) Starting with best initial solution is found in part (c), obtain an optimal solution, and hence produce transportation schedule.

Question 7

Three factories F_1, F_2, F_3 supply four supermarkets S_1, S_2, S_3 and S_4 with tea. $F_1, F_2,$ and F_3 produce 8, 10, and 9 lorry-loads of tea per day respectively. S_1, S_2, S_3 and S_4 order 6, 5, 8 and 8 lorry-loads of tea per day respectively. The transportation costs (in pounds per lorry-load) are given in Table 2; so for example it costs.\$7 to transport a lorry-load from F_2 to S_2 .

- (i) Determine the minimum cost delivery plan, and find its cost.
(ii) If the transportation cost for shipping 1 unit from F_3 to S_2 is decreased by 3 units what will be the minimum cost?
(iii) If the cost of supplying a unit from F_2 to S_1 is increased to 5 find the minimum transportation cost.

	S_1	S_2	S_3	S_4
F_1	6	4	5	6
F_2	3	7	2	1
F_3	5	6	3	4

Question 8

Consider the following problem

$$\text{Maximize } Z = -5x_1 + 5x_2 + 13x_3$$

Subject to;

$$-x_1 + x_2 + 3x_3 \leq 20 \longrightarrow (1)$$

$$12x_1 + 4x_2 + 10x_3 \leq 90 \longrightarrow (2)$$

$$x_1, x_2, x_3 \geq 0$$

If we let x_4 and x_5 be the slack variables for the constraints respectively, the simplex method yields the following final set of equations.

$$\begin{aligned} Z + 2x_3 + 5x_4 &= 100 \\ -x_1 + x_2 + 3x_3 + x_4 &= 20 \\ 16x_1 - 2x_3 - 4x_4 + x_5 &= 10 \end{aligned}$$

- (i) Interpret this solution assuming x_1, x_2, x_3 , are 3 products and Z being the profit.
- (ii) Change the right-hand side of constraint 1 to 30. What will happen to the current optimal solution? (Do not optimize corresponding to this change)
- (iii) Change the Right Hand Side of the original problem to $\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} 10 \\ 100 \end{bmatrix}$ and find the new optimal solution.
- (iv) Introduce a new constraint $2x_1 + 3x_2 + 5x_3 \leq 50$. (Denote its slack variable by x_6) Using an appropriate tableau format, show and explain what will happen to the current solution but do not re-optimize.

Question 9

Consider the following Linear programming Problem (LPP).

$$\text{Maximize } Z = 2x_1 - 3x_2$$

Subject to;

$$x_1 + x_2 \geq 3 \longrightarrow (1)$$

$$3x_1 + x_2 \leq 6 \longrightarrow (2)$$

$$x_1, x_2 \geq 0$$

- a. You are told that the optimal solution is $x_1 = 3/2, x_2 = 3/2$. Verify this Statement by duality.
- b. State two LPP forms that can be obtained by modifying the problem in such a way that the Dual Simplex method can be used in solving them.
- c. Solve one of the above forms obtained in (b) by Dual Simplex method.

Question 10

Table 1 is a transportation tableau, used in identifying a network which minimizes the total transportation cost.

	D ₁	D ₂	D ₃	D ₄	
F ₁	9	7	12	8	18
F ₂	15	12	12	15	4
F ₃	8	9	6	12	6
F ₄	14	12	11	12	12
	6	14	15	5	40

Table 1

- (i) Find the initial basic feasible solution by Vogel's Approximation method.
- (ii) Find the optimal transportation schedule.
(Duplicating tables is not necessary; do as many as calculations on the same table.)
- (iii) Does this problem have any alternative solutions? If "Yes" find one such solution.
- (iv) Suppose that C_{43} is increased from 11 to 13. Is the solution still optimal? If not, find the new optimal solution.

Question 11

The following table gives data on normal time and cost and crash time and cost for a project.

1. Draw the network and identify the critical path.
2. What is the normal project duration and associated cost.
3. Find out total float for each activity.
4. Crash the relevant activities systematically and determine the optimal time and cost.

Activity	Time estimate (weeks)		Direct cost estimate (Rs. 1000)	
	Normal	Crash	Normal	Crash
1-2	3	2	30	400
2-3	3	3	30	30
2-4	7	5	420	580
2-5	9	7	720	810
3-5	5	4	250	300
4-5	0	0	0	0
5-6	6	4	320	410
6-7	4	3	400	470
6-8	13	10	780	900
7-8	10	9	1000	1200

Indirect cost is Rs.50 per week.

Question 12

You are given the information on the durations and direct and indirect costs involved in a nine activity project.

Activity	Normal Schedule		Crash Schedule	
	Time	Cost'000	Time	Cost'000
1-2	5	8	3	20
1-3	6	12	5	15
2-3	3	12	1	22
2-4	8	10	5	22
3-5	2	5	1	12
3-6	11	40	8	46
4-5	0	0	0	0
4-6	1	4	1	4
5-6	12	8	9	11

You will acquire an indirect cost of Rs. 5000 per week.

- (i) Construct the network diagram and find the critical path.
- (ii) Find the normal cost and the normal project completion time.
- (iii) Give the optimum cost schedule with the durations for each activity.

Question 13

A firm has three plants that supply commodity to three regions. The unit transportation costs from each plant to each region are in upper left hand corners of the table below. Plant capacities and demand at regions are also given in the margins.

Region \ Plant	W	N	S	Capacity
P ₁	20	10	40	200
		100		
P ₂	10	20	30	300
	100		100	
P ₃	10	10	20	100
			100	
Demand	100	100	200	

The current transportation schedule is given in the above table.

- (i) Is there a better solution than given above? If so, find it.
- (ii) Suppose the demand at the region *W* is increased by 20% , what is the effect of it on the transportation schedule?
- (iii) What would be the saving in transportation cost if the capacity at the third plant (P₃) is doubled?

Question 14

A small electronics dealer buys various components to assemble them into transistors, tape recorders and small stereo sets. He does the assembly work after business hours and in the week ends for sale during the next week. In a week the dealer has time to assemble at most 500 kits of any one or the combined items. Transistors and tape recorders have a weekly combined order of at least 150 units. Transistors being very popular the number of these units assembled must exceed the number of the combination of tape recorders and stereos by 100 units. He has further found that every transistor contributes Rs. 75, a tape recorder Rs. 125 and a stereo set Rs. 150 to the profit. Suggest him a production mix to run in the business showing that your decision leads to the maximum profit under the given situation.

Question 15

An air conditioning manufacturer produces room air conditioners at plants in Houston, Phoenix, and Memphis. These are sent to regional distributors in Dallas, Atlanta, and Denver. The shipping cost: vary and the company would like to find the least-cost way to meet the demands at each of the distribution centers. Dallas needs to receive 800 air conditioners per month, Atlanta needs 600, and Denver needs 200. Houston has 850 air conditioners available each month. Phoenix has 650 and Memphis has 300. The shipping cost per unit from Houston to Dallas is \$8, to Atlanta is \$12 and to Denver is \$10. The cost per unit from Phoenix to Dallas is \$10, to Atlanta is \$14 and to Denver is \$9. The cost per unit from Memphis to Dallas is \$11, to Atlanta is \$8, and to Denver is \$12. How many units should be shipped from each plant to each regional distribution center? What is the total cost for this?