Duality in Linear Programming

Introduction

Every LPP (called the primal) is associated with another LPP (called its dual). Either of the problem can be considered as primal with the other one as dual. The importance of the duality concept is due to two main reasons:

(i) If the primal contains a large number of constraints and a smaller number of variables, the labor of computation can be considerably reduced by converting it in to the dual problem and then solving it.

(ii) The interpretation of the dual variables from the cost or economic point of view, proves extremely useful in making future decisions in the activities being programmed

Formation of dual problems

For formulating dual problem, first we bring the problem in the canonical form. The following changes are used in formulating the dual problem.

(1) Change the objective function of maximization in the primal into minimization one in the dual and vice versa.

(2) The number of variable in the primal will be the number of constraints in the dual and vice versa.

(3)The cost coefficients $C_I, C_2 \dots \mathbf{C}_n$ in the objective function of the primal will be the rhs constant of the constraints in the dual and vice versa.

(4) in forming the constraints for the dual, we consider the transpose of the body matrix of the primal problem.

(5) The variables in both problems are non-negative.

(6) If the variable in the primal is unrestricted in, sign, then the corresponding constraint in the dual will be an equation and vice versa.

Definition of the dual problem

let the primal problem be

maximize $z = c_1x_1 + c_2x_2 + \dots + c_nx_n$

Subject to:

$$a_{11}x_1 + a_{12}x_2 + \dots + a_{1n}x_n \leq b_1$$
$$a_{21}x_1 + a_{22}x_2 + \dots + a_{2v}x_n \leq b_2$$

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$$a_{m1}x_1 + a_{m2}x_2 + \dots + a_{mn}x_n \leq b_m$$
$$x_1, x_2, \dots x_n \geq 0$$

The dual problem is define as

Min $z' = b_1y_1 + b_2y_2 + \dots + b_ny_n$

Subject to:

$a_{11}y_1 + a_{21}y_2 + ... + a_{m1}y_n \geq c_1$

$a_{12}y_1 + a_{22}y_2 + ... + a_{m2}y_n \geq c_2$

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$a_{1m}y_1 + a_{2m}y_2 + ... + a_{mn}y_n \geq c_m$

$y_1, y_2, ... y_n \geq 0$

Where $y_1, y_2, ... y_n$ are called dual variables.

Important Results in Duality

1. The dual of the dual is primal. .
2. If one is a maximization problem then the other is a minimization one.
3. The necessary and sufficient condition for any LPP and its dual to have an optimal solution is that both must have feasible solution.
4. Fundamental duality theorem states if either the primal or dual problem has a finite optimal solution, then the other problem also has a finite optimal solution and also the optimal values of the objective function in both the problems are the same ie max z=min z^0. The solution of the other problem can be read from the z. objective row below the columns of slack, surplus variables.
5. Existence theorem states that, if either problem has an unbounded solution then the other problem has no feasible solution.
6. Complementary slackness theorem: according to which

(i) If a primal variable is positive, then the corresponding dual constraint is an equation at the optimum and vice versa.

(ii) If a primal constraint is a strict inequality then the corresponding dual variable is zero at the optimum and vice versa. .