Examples 2

1. The weights, in grams, of a consignment of apples are normally distributed with mean μ and standard deviation 4. A sample of size 25 is taken and the statistics R and T are calculated as follows:

   $$R = X_{25} - X_1 \text{ and } T = X_1 + X_2 + \ldots.. + X_{25.}$$

   Find the distributions of R and T.

2. A large bag contains counters. Sixty per cent of the counters have the number 0 on them and 40% have the number 1.

   (a) Find the mean μ and the variance σ^2 for this population of counters.

   A random sample of size 3 is taken from this population.

   (b) List all possible samples.

   (c) Find the sampling distribution for the mean $\overline{X} = \dfrac{X_1 + X_2 + X_3}{3}$ where X_1, X_2 and X_3 are the three variables representing samples 1, 2 and 3.

   (d) Hence find E ($\overline{X}$) and Var($\overline{X}$).Find the sampling distribution for the mode M.

   (f) Hence find E (M) and Var(M).

3. The lengths of nails produced by a certain machine are normally distributed with a mean μ and standard deviation σ. A random sample of 10 nails is taken and their lengths $\{X_1, X_2, X_3, X_4, X_5\}$ are measured.

   (a) Write down the distributions of the following:

   (i) $\sum_{1}^{10} X_i$ (ii) $\dfrac{2X_1 + 3X_{10}}{5}$ (iii) $\sum_{1}^{10}(X_i - \mu)$

   (iv) $\overline{X}$ (v) $\sum_{1}^{10} X_i - \sum_{6}^{10} X_i$ (vi) $\sum_{1}^{10}(\dfrac{X_i - \mu}{\sigma})$

   (b) State which of the above are statistics.

4. A machine operator checks a random sample of 20 bottles from a production line in order to estimate the mean volume of bottles (in cm^3) from this production run. The 20 values can be summarized as $\sum x = 1300$ and $\sum x^2 = 84685$.

   (a) Use this sample to find unbiased estimates of μ and σ^2.

(b) A supervisor knows from experience that the standard deviation of volumes on this process, σ, should be 3 cm^3 and he wishes to have an estimate of μ that has a standard error of less than 0.5 cm^3.

What size sample will he need to achieve this?

(c) The supervisor takes a further sample of size 16 and finds $\sum x = 1060$.

Combine the two samples to obtain a revised estimate of μ.

5. An electrical company repairs very large numbers of television sets and wishers to estimate the mean time taken to repair a particular fault.

It is known from previous research that the standard deviation of the time taken to repair this particular fault is 2.5 minutes.

The manager wishes to ensure that the probability that the estimate differs from the true mean by less than 30 seconds is 0.95. Find how large a sample is required.

6. A large bag of coins contains 1p, 5p, and 10p coins in the ratio 2: 2: 1.

(a) Find the mean μ and σ^2 for the variance for the values of coins in this population.

(b) A random sample of two coins is taken and their values X_1 and X_2 are recorded.

List all the possible samples.

(c) Find the sampling distribution for the mean $\overline{X} = \dfrac{X_1 + X_2}{2}$

(d) Hence show that E ($\bar{X}$) = μ and Var ($\bar{X}$) = $\dfrac{\sigma^2}{n}$

7. A manufacturer of self-assembly furniture required bolts of two lengths, 5 cm and 10 cm, in the ratio 2: 1.

(a) Find the mean μ and the variance σ^2 for the lengths of bolts in this population.

A random sample of three bolts is selected from a large box containing bolts in the required ratio.

(b) List all possible samples.

(c) Find the sampling distribution for the mean $\bar{X}$

(d) Hence find E($\bar{X}$) and Var ($\bar{X}$).

(e) Find the sampling distribution for the mode M.

(f) Hence find E(M) and Var (M).

(g) Find the bias when M is used as an estimator of the population mode.