Examples 7

1. The weight of jam in a jar, measured in grams, is distributed normally with a mean of 150 and a standard deviation of 5. The production process occasionally leads to a change in the mean weight of jam per jar but the standard deviation remains unaltered.
   The manager monitors the production process. For every new batch she takes a random sample of 25 jars and weighs their contents to see if there has been any reduction in the mean weights of jam per jar.
   Find the critical values for the test statistics $\overline{X}$, the mean weight of jam in a sample of 25 jars, using
   (a) a 5% level of significance,
   (b) a 1% level of significance.
   Given that the true value of μ for the new batch is in fact 147,
   (c) Find the probability of a type II error for each of the above critical regions.

2. Bags of sugar having a nominal weight of 1 kg are filled by a machine. From past experience it is known that the weight, X kg, of sugar in the bags is normally distributed with a standard deviation of 0.04 kg. At the beginning of each week a random sample of 10 bags is taken in order to see if the machine needs to be reset. A test is then carried out at the 5% significance level with $H_0 : \mu = 1.00$ kg and $H_1 : \mu \neq 1.00$ kg.
   (a) Find the critical region for this test.
   (b) Given that the sample taken has a mean of 1.02 kg, test whether or not the mean has changed.
   (c) Assuming that the mean weight has in fact changed to 1.02 kg, find the type I and type II errors for the test.

3. The treatment for a certain illness has a probability of success of 0.45. A new treatment is being researched and, in a trial of 20 people, 13 were successfully treated using the new drug.
   (a) Conduct an appropriate hypothesis test to examine whether or not there is enough evidence at the 5% significance level to suggest that the new drug is an improvement over the old one. Choose your critical region so that the probability of an observation lying in the critical region is ≤ 0.05.
   (b) Calculate the probability of type I error.
   Further research on the new drug showed that the true probability of a successful cure was 0.5.
   (d) Use the information given above to calculate the probability of a type II error having been made in part (a).

4. Accidents occur on a stretch of motorway at a rate of 6 per month. Many of the accidents that occur involve vehicles skidding into the back of other vehicles. By way of a trial, a new type of road surface, said to reduce the risk of vehicles skidding, is laid on this stretch of road. During the first month of operation 4 accidents occur.
   (a) Test whether or not there is evidence that there has been an improvement. Use a 5% level of significance.
   (b) Calculate the type I error of this type.
   (c) If the true rate of accidents occurring with this type of road surface was 3.5, calculate the probability of type II error.

5. A coin is tossed 20 times and a head is obtained on 7 occasions.
   (a) Test to see whether or not the coin is biased.
   (b) Calculate the type I error for this test.
   (c) Given that the coin is biased and that this bias causes the head to appear 3 times for each tail that appear, calculate the type II error for the test.

6. In a binomial experiment consisting of 10 trials the random variable X represents the number of successes and p the probability of success.
   In a test of $H_0 : p = 0.4$ against $H_1 : p > 0.4$, a critical region of $X \geq 8$ was used.
   Find the power of this test when
   (a) $p = 0.5$,
   (b) $p = 0.8$.
   (c) Comment on your results.