

MODEL QUESTIONS & ANSWERS MA(101)
3 DIMENSIONAL GEOMETRY

Q(1). Angle between the lines is (i) $\theta = \cos^{-1}\left(-\frac{1}{6}\right)$ (ii) $\theta = \frac{\pi}{2}$, (iii) $\theta = \cos^{-1}\left(\frac{4}{21}\right)$

Q(2). $\underline{a} = 4\underline{i} + 6\underline{j} + 2\underline{k}$, $\underline{b} = 3\underline{i} + 10\underline{j} + 5\underline{k}$, and $\underline{c} = -4\underline{i} + 5\underline{j} + 5\underline{k}$

$(\underline{a} \times \underline{b}) \cdot \underline{c} = \begin{vmatrix} 4 & 6 & 2 \\ 3 & 10 & 5 \\ -4 & 5 & 5 \end{vmatrix} = 0$ Therefore, $(\underline{a} \times \underline{b}) \cdot \underline{c} = 0$ implies that these four points are

coplanar.

Q(3). Equation of the plane is $9x + 8y - 3z = 38$.

Q(4). Equations of the plane are (i) $x + 5y - 6z = -19$. (ii) $5x - 6y + 7z = 20$,

(iii) $x + y + z = 9$.

Q(5). Equation of the plane is $71x + 19y - 16z = 57$.

Q(6). (i) Equation of st : line $\frac{x-1}{4} = \frac{y-2}{-2} = \frac{z-3}{0} = t$, a vector parallel to the line is

$\underline{n} = 4\underline{i} - 2\underline{j}$. Therefore $\underline{n} \cdot \underline{k} = 0$, hence line is normal to z axis.

(ii) Distance between two points is 13.

Q(7). Equation of st : line $\frac{x+1}{1} = \frac{y-3}{2} = \frac{z-2}{2} = t$, coordinates of the foot is $\left(\frac{-5}{3}, \frac{5}{3}, \frac{2}{3}\right)$

Image point is $\left(\frac{34}{11}, \frac{65}{11}, \frac{-8}{11}\right)$.

Q(9). Equations of bisectors through lines are $\frac{x-3}{38} = \frac{y+4}{-49} = \frac{z-5}{-17}$ and

$$\frac{x-3}{14} = \frac{y+4}{23} = \frac{z-5}{-35}$$

Q(10). Symmetrical equation of line of projection is $\frac{x-1/3}{10} = \frac{y}{78} = \frac{z+1/3}{2}$

Q(11) (i) Equation of plane is $-3x + y + 4z = 11$. (ii) Angle between lines is

$$\theta = \cos^{-1}\left(\frac{7}{\sqrt{14 \times 101}}\right)$$

Q(14). Equation of st : line $\frac{x}{4} = \frac{y}{21} = \frac{z}{-7} = t$.

Model Questions With Solutions

Q(1). Find the equations of the line joining (1,2,3) and (-3,4,3) and show that it is perpendicular to Z axis

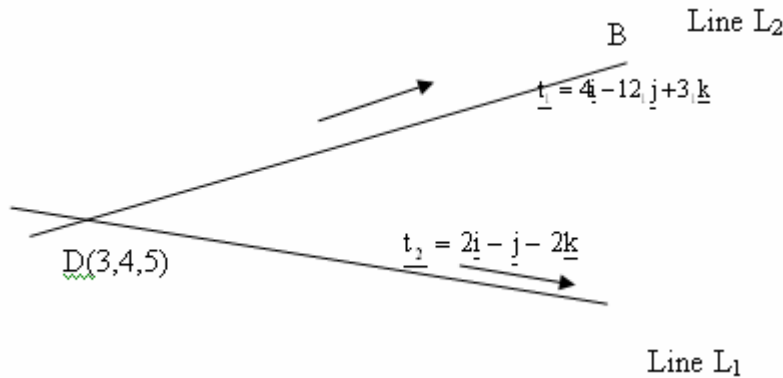
Solution: Equation of st : line $\frac{x-1}{4} = \frac{y-2}{-2} = \frac{z-3}{0} = t$, a vector parallel to the line is

$\underline{n} = 4\underline{i} - 2\underline{j}$. Therefore $\underline{n} \cdot \underline{k} = 0$, hence line is normal to z axis.

Q(2). Find the equation of the line bisecting the angle between the lines

$$\frac{x-3}{2} = \frac{y+4}{-1} = \frac{z-5}{-2}, \quad \frac{x-3}{4} = \frac{y+4}{-12} = \frac{z-5}{3}$$

Solution: Let \underline{t}_1 and \underline{t}_2 be parallel vectors to the given two lines L_1 and L_2 respectively.



$$\underline{t}_1 = 4\underline{i} - 12\underline{j} + 3\underline{k} \quad \text{and} \quad \underline{t}_2 = 2\underline{i} - \underline{j} - 2\underline{k}$$

Therefore unit vector parallel to the same lines are $\frac{2\underline{i} - \underline{j} - 2\underline{k}}{13}$ and $\frac{4\underline{i} - 12\underline{j} + 3\underline{k}}{3}$

Hence vector along the lines of bisectors

$$\frac{2\underline{i} - \underline{j} - 2\underline{k}}{13} \pm \frac{4\underline{i} - 12\underline{j} + 3\underline{k}}{3} \quad \text{i.e.,} \quad \frac{38\underline{i} - 49\underline{j} - 17\underline{k}}{39} \quad \text{and} \quad \frac{14\underline{i} + 23\underline{j} - 35\underline{k}}{39}$$

\therefore Equations of bisectors through lines are $\frac{x-3}{38} = \frac{y+4}{-49} = \frac{z-5}{-17} = t$ and

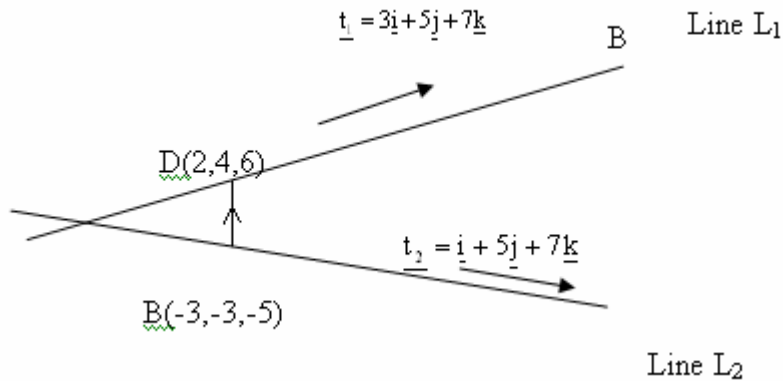
$$\frac{x-3}{14} = \frac{y+4}{23} = \frac{z-5}{-35} = k.$$

Q(3) Show that the lines $\frac{x+3}{3} = \frac{y+3}{5} = \frac{z+5}{7}$ and $\frac{x-2}{1} = \frac{y-4}{5} = \frac{z-6}{7}$ intersect and

find the co-ordinates of the point of intersection

Solution

Suppose two lines L_1 and L_2 intersect, then lines L_1 and L_2 are coplanar lines.



Let \underline{t}_1 and \underline{t}_2 be parallel vectors to the given two lines L_1 and L_2 respectively.

Therefore \underline{t}_1 , \underline{t}_2 and

\overrightarrow{BD} are also coplanar vectors. Where $\overrightarrow{BD} = 5\underline{i} + 7\underline{j} + 11\underline{k}$

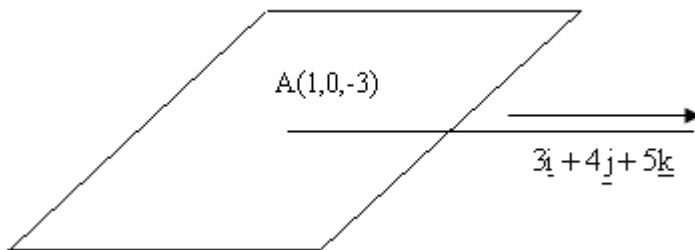
$$\Rightarrow (\underline{t}_1 \times \underline{t}_2) \cdot \overrightarrow{BD} = 0$$

$$\begin{vmatrix} 3 & 5 & 7 \\ 1 & 5 & 7 \\ 5 & 7 & 11 \end{vmatrix} \neq 0 \therefore \text{lines } L_1 \text{ and } L_2 \text{ are not coplanar and point of intersection does not}$$

exists.

Q(4). Find the equations of the perpendicular from $(1, 0, -3)$ to the line

$$\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$$



Solution Let a line through (1,0,3) which is perpendicular to the vector $3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$ can

be written as in the form $\frac{x-1}{l} = \frac{y-0}{m} = \frac{z+3}{n} = k$ with $l\mathbf{i} + m\mathbf{j} + n\mathbf{k}$ normal to

$$3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$$

$\therefore 3l + 4m + 5n = 0$ One can choose arbitrarily l, m, n values so that $3l + 4m + 5n = 0$

i.e, $l=4, m=-3, n=0$ or $l=2, m=6, n=-6$

$$\frac{x-1}{4} = \frac{y-0}{-3} = \frac{z+3}{0} = t \quad \text{or} \quad \frac{x-1}{2} = \frac{y-0}{6} = \frac{z+3}{-6} = k$$

Q(5). Find the co-ordinates of the foot of the perpendicular from P(1, 2, 3) to the line

$$\frac{x-2}{1} = \frac{y-1}{2} = \frac{z}{3} \quad \text{Find the length of the perpendicular and its equations}$$

Solution: Coordinates of the foot of the perpendicular $\left(\frac{19}{7}, \frac{17}{7}, \frac{15}{7}\right)$

Q(6) Find the points on the lines $\frac{x-6}{3} = \frac{y-7}{-1} = \frac{z-4}{1}$ and $\frac{x}{-3} = \frac{y+9}{2} = \frac{z-2}{4}$ which are

nearest to each other. Hence find the shortest distance between the lines and its equation..

Solution: coordinates of the nearest points are (3, 8, 3), (-3, -7, 6), shortest distance =

$$\sqrt{270}, \quad \text{Equation of the shortest distance is } \frac{x-3}{6} = \frac{y-8}{15} = \frac{z-3}{3} = k.$$

Q(7). Find the shortest distance between the lines $\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1}$ and

$$\frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

Solution shortest distance between two lines = $2\sqrt{29}$,

Q(8). Prove that the shortest distance between the diagonal of the rectangular

parallelepiped and the edges not meeting it are $\frac{bc}{\sqrt{b^2+c^2}}, \frac{ca}{\sqrt{a^2+c^2}}, \frac{ab}{\sqrt{b^2+a^2}}$ where

$a, b,$ and c are length of the edges.

Q(9). Find a vector normal (perpendicular) to the plane of P(1,-1, 0), Q(2, 1,-1) and R(-1, 1, 2).

Solution A vector normal to the plane is $6\mathbf{i} + 6$.

Q(10). Find the vector equation of the line through the point with position vector $2\mathbf{i} - \mathbf{j} - 3\mathbf{k}$ which is parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$. Determine the points corresponding to $\lambda = 3, 0, -2$ in the resulting equation

Solution A vector equation of the line is $\mathbf{r} = 2\mathbf{i} - \mathbf{j} - 3\mathbf{k} + \lambda(\mathbf{i} + \mathbf{j} + \mathbf{k})$ and the corresponding points are