UNIVERSITY OF MORATUWA

MSC/POSTGRADUATE DIPLOMA IN OPERATIONAL RESEARCH

MA(5001) INTRODUCTION TO STATISTICS
THREE HOURS **AUGUST 2009**

Answer **FIVE** questions and **NO MORE**.

ADDITIONAL MATERIAL:

Statistical Tables will be available.

INSTRUCTIONS TO CANDIDATES:

This paper contains 8 questions and 8 pages.

Answer **FIVE** questions and NO MORE.

This is a closed book examination.

This examination accounts for 70% of the module assessment

Assume reasonable values for any data not given in or with the examination paper. Clearly State such assumptions made on the script.

If you have any doubt as to the interpretation of the wording of a question, make your own decision, but clearly state it on the script.

Continued……

NO QUESTIONS

Question 1

(a). Let Y possess a probability density function such that;

$$f(y)=\begin{cases} c(2-y), & 0 \le y \le 2 \\ 0, & \text{elsewhere,} \end{cases}$$

(i). Find c.

(ii). Find F(y).

(iii). Graph f(y) and F(y).

(iv) Use F(y) in (ii) to find $p(1 \le y \le 2)$.

(v).Use the geometric figure for *f(y)* to calculate $p(1 \le y \le 2)$.

(b). Two refills for a ballpoint pen are selected at random from a box that contains 3 blue refills, 2 red refills, and 3 green refills. If X is the number of blue refills and Y is the number of red refills selected. Show that the following joint probabilities are satisfied as given in the Table 1.

Hence, find

(i) the marginal probability functions of X and Y.

(ii) the expected values of X and Y.

(iii) the covariance of X and Y.

(vi) $p(X+Y \le 1)$.

Table 1 Joint Probability Distribution of X, and Y

f(x,y)		x		Row Totals
	0	1	2	
0	$\frac{3}{28}$	$\frac{9}{28}$	$\frac{3}{28}$	$\frac{15}{28}$
y 1	$\frac{3}{14}$	$\frac{3}{14}$		$\frac{3}{7}$
2	$\frac{1}{28}$			$\frac{1}{28}$
Column Totals	$\frac{5}{14}$	$\frac{15}{28}$	$\frac{3}{28}$	1

Question 2

(a) An advertising agency, indicates that matters of taste cannot be ignored in television advertising. Based on a mail survey of 3440 people, 40% indicated that they found TV commercials to be in poor taste 55% say that they avoid products whose commercials were judged to be in poor taste and of this latter group, only 20% ever complained to a TV station or an advertiser about their dissatisfaction.

(i) Find a 95 % confidence interval for the percentage of TV viewers who find TV commercials to be in poor taste.

(ii) Find a 95 % confidence interval for the percentage of TV viewers who avoid products which use TV commercials that they consider to be in poor taste.

(iii) Find a 95 % confidence interval for the percentage of those who avoid products who have complained to the TV station or the advertiser about poor taste in a TV commercial.

(b) The administrators for a hospital wished to estimate the average number of days required for treatment of patients between the ages of 25 and 34. A random sample of 500 hospital patients between these ages produced a mean and standard deviation equal to 5.4 and 3.1 days, respectively. Construct a 95 % confidence interval for the mean length of stay for the population of patients from which the sample was drawn.

Question 3

The manager of a construction job needs to figure prices carefully before submitting a bid. He also needs to account for uncertainty (variability) in the amounts of products he might need. To oversimplify the real situation, suppose a project manager decides that the amount of sand, in yards, needed for a construction project is a random variable Y_1, which is normally distributed with a mean of 10 yards and a standard deviation of 0.5 yard. The amount of cement mix needed, in hundreds of pounds, also is a random variable, Y_2, which is normally distributed with a mean of 4 and a standard deviation of 0.2. The sand costs \$7 per yard and the cement mix costs \$3 per hundred pounds. Adding \$100 for other costs, he computes his total cost to be

$$U = 100 + 7Y_1 + 3Y_2 .$$

If Y_1 and Y_2 are independent, how much should the manager bid in order to ensure that the true costs will exceed the amount bid with a probability of only .01? Is the independence assumption reasonable here?

(b) Bolts that are used in the construction of an electric transformer are supposed to be 0.060 inches in diameter, and any bolt with diameter less than 0.058 inches or greater than 0.062 inches must be scrapped. The machine that makes these bolts is set to produce bolts of 0.060 inches in diameter, but it actually produces bolts with diameters following a normal distribution with $\mu = 0.060$ inches and $\sigma = 0.001$ inches. Find the proportion of bolts that must be scrapped.

Question 4

(a). The employees of firm were surveyed and then classified by type of work and sex. The results were tabulated.

Table2

Sex	**Type of Work**			
	Production	Office	Sale	Total
Female	12	25	13	50
Male	188	475	287	950
Total	200	500	300	1000

(i). What is the probability that an employee who is randomly selected is a female working in production?

(ii). What is the probability that an employee who is randomly selected works in production?

(iii). What is the probability that an employee who is randomly selected being female given that the employee works in problem?

(iv). Find the marginal probability distribution of type work?

(b)Suppose that a unit of mineral ore contains a proportion Y_1 of metal A and a proportion Y_2 of metal B. Experience has shown that the joint probability density function of (Y_1, Y_2) is uniform over the region $0 \le y_1 \le 1,\ 0 \le y_2 \le 1,\ 0 \le y_1 + y_2 \le 1,$. Let $U = Y_1 + Y_2$ be the proportion of metals A and B per unit.

(i) Find the probability density function for U.

(ii) Find E(U) by using the answer to part (i).

(iii) Find E(U) by using only the marginal densities of Y_1 and Y_2.

Question 5

(a)Three balanced coins are tossed independently. One of the variables of interest is Y_1 = the number of heads. Let Y_2 denote the amount of money won on a side bet in the following manner. If the first head occurs on the first toss, you win \$1. If the first head occurs on toss 2 or on toss 3 you win \$2 or \$3, respectively. If no heads appear, you lose \$1 (that is, win -\$1).

(i) Find the joint probability function for Y_1 and Y_2.

(ii) What is the probability that less than three heads occur and you win \$1 or less? [That is, find F(2, 1)].

(b) Let Y_I denote the weight (in tons) of a certain bulk item stocked by a supplier at the beginning of a week and suppose that Y_I has a uniform distribution over the interval $0 \le y_1 \le 1,$. Let Y_2 denote the weight of this item sold by the supplier during the week and suppose that Y_2 has a uniform distribution over the interval $0 \le y_1 \le 1,$ where y_1 is a specific value of Y_1.

(i)Find the joint density function for y_1 and y_2.

(ii) If the supplier stocks a half ton of the item, what is the probability that she sells more than a quarter ton?

(iii)If it is known that the supplier sold a quarter ton of the item, what is the probability that she had stocked more than a half ton?

Question 6

A company dealing with newly invented telephonic device is faced with the problem of selecting the following strategies:

(i) manufacture the device itself;

(ii) to be paid on a royalty basis by another manufacturer,

(iii) sell the rights for its invention for a lump sum.

The profit in thousands of rupees that can be expected in each case and the probabilities associated with the sales volume are shown in the following table:

Table 3

Event	Probability	Manufacture itself	Royalties	Sell the rights
High demand	0.2	100	40	20
Medium demand	0.3	30	25	20
Low demand	0.5	-10	15	20

(a) Represent the company's problem in the form of a decision tree.

(b) Extend the diagram further for the following additional information:

(i) If the company manufactures itself and sales are medium or high, it has the opportunity of developing a new version of its telephone.

(ii) From past experience, it estimates that there is 60% chance of successful development.

(iii) If the cost of development is Rs. 20 and the returns (after deducting development cost) are Rs. 35 and Rs. 10 for high and medium demand respectively.

Question 7

(a) An efficiency expert claims that by introducing a new type of machinery into a production process he can decrease substantially the time required for production. Because of the expense involved in maintenance of the machines, management feels that unless the production time can be decreased by at least 8.0% they cannot afford to introduce the process. Six resulting experiments show that the time for production is decreased by 8.4% with standard deviation of 0.32%. Using a level of significance of 0.05, test the hypothesis that the new machine should be introduced.

(b) To test the effects of a new fertilizer on wheat production, a tract of land was divided into 60 squares of equal areas, all portions having identical qualities as to soil, exposure to sunlight, etc. The new fertilizer was applied to 30 squares and the old fertilizer was applied to the remaining squares. The mean number of kilograms of wheat harvested per square of land using the new fertilizer was 496.31 with a standard deviation of 17.18 kilograms. The corresponding mean and standard deviation for the squares using the old fertilizer were 485.41 and 14.73 kilograms respectively. Using a significance level of 0.05, test the hypothesis that the new fertilizer is better than the old one.

Question 8

The following data represent the chemistry grades for a random sample of 12 freshmen at a certain college along with their scores on an intelligence test administered while they were still seniors in high school:

Table 4

Student	Test Score(X)	Chemistry Grade(Y)
1	65	85
2	50	74
3	55	76
4	65	90
5	55	85
6	70	87
7	65	94
8	70	98
9	55	81
10	70	91
11	50	76
12	55	74

(a) Compute and interpret the sample correlation coefficient.

(b) Test the hypothesis that $\rho = 0.5$ against the alternative that $\rho > 0.5$. Use $\alpha = 0.05$.

(c) Find the least squares of simple linear regression line of y on x ($y = \alpha + \beta x + e$).

(d) Find 95% confidence interval of β .

(e) Check whether β is significant different from zero?