UNIVERSITY OF MORATUWA

MSC/POSTGRADUATE DIPLOMA IN FINANCIAL MATHEMATICS

MA 5101 OPERATIONAL RESEARCH TECHNIQUE I

THREE HOURS **October 2008**

Answer **FIVE** questions and **NO MORE**.

Question 1

ALI Electronics Incorporated manufactures the following six microcomputer peripheral devices: internal modems, external modems, Circuit boards, CD drives, hard drives, and memory boards. Each of these technical products requires time, in minutes, on three types of electronic testing equipment, as shown in the Table 1.

The first two test devices are available 120 hours per week. The third (device 3) requires more preventive maintenance and may be used only 100 hours each week. The market for all six computer components is vast, and ALI Electronics believes that it can sell as many units of each product as it can manufacture. The Table 2 that follows summarizes the revenues and material costs for each product:

In addition, variable labor costs are $15 per hour for test device 1, $12 per hour for test device 2, and $18 per hour for test device 3. ALI Electronics wants to maximize its profits.

(a) Formulate this problem as an LP model.

(b) Solve the problem by computer. What is the best product mix?

(c) What is the value of an additional minute of time per week on test device 1? Test device 2? Test device 3? Should ALI Electronics add more test device time? If so, on which equipment?

TABLE 1

	Internal Modem	External Modem	Circuit Board	CD Drivers	Hard Drivers	Memory Board
Test device 1	7	3	12	6	18	17
Test device 2	2	5	3	2	15	17
Test device 3	5	1	3	2	9	2

TABLE 2

Device	Revenue Per Unit Sold ($)	Material Cost Per Unit ($)
Internal modem	200	35
External modem	120	25
Circuit board	180	40
CD drive	130	45
Hard disk drive	430	170
Memory board	260	60

Question 2

(a)Consider the following optimal tableau, where S_1, S_2 and S_3 are slack variables added to the original problem is given in Table 3.

(i) What are the shadow prices for the three constraints? What does a zero shadow price mean? How can this occur?

(ii) How much could the right-hand side of the first constraint be changed without changing the solution.?

(ii) How much could the right-hand side of the third constraint be changed without changing the solution mix?

(iii) What would the optimal solution be if the profit on X_2 were changed to $115 instead of $120?

(iv) What would the optimal solution be if the profit on X_I were changed to $100 instead of $90? How much would the maximum profit change?

TABLE 3 Optimal Simplex Tableau

Basic	X_I	X_2	X_3	S_1	S_2	S_3	quantity
X_2	-1.5	1	0	0.125	-0.75	0	37.5
X_3	3.5	0	1	-0.125	1.25	0	12.5
S_3	-1.0	0	0	0	-0.5	1	10.0
Z	55	0	0	3.75	22.5	0	5,625

Question 4

A company wishes to determine an investment strategy for each of the next four years. Five investment types have been selected, investment capital has been allocated for each of the coming four years and maximum investment levels have been established for each investment type. An assumption is that amounts invested in any year will remain invested until the end of the planning horizon of four years. The Table 4 summarizes the data for this problem. The values in the body of the table represent net return on investment of one rupee upto the end of the planning horizon. For example, a rupee invested in investment type B at the beginning of year 1 will grow to Rs. 1.90 by the end of the fourth year, yielding a net return of Re. 0.90.

TABLE 4

Investment made at the beginning of the year	Net return data on investment type					Rupees available (in000)
	A	B	C	D	E	
1	0.80	0.90	0 60	0.75	1.00	500
2	0.55	0.65	0.40	0.60	0.50	600
3	0.30	0.25	0.30	0.50	0.20	750
4	0.15	0.12	0. 25	0.35	0.10	800
Maximum Rupees investment (in 000)	750	600	500	800	1,000	

The objective in this problem is to determine the amount to be invested at the beginning of each year in each investment to maximize the net rupee return for the four-year period.

Question 5

Consider the following transshipment problem involving 4 sources and 2 destinations The supply values of the sources S_1, S_2, S_3 and S_4 are 100 units, 200 units, 150 units and 350 units respectively. The demand values of destinations D, and D, are 350 units and 450 units, respectively The transportation cost per unit between different sources and destinations are summarized as in Table 5. Solve the transshipment problem.

TABLE 5 C_{ij} Values for Question 5

Destination

Source		S_1	S_2	S_3	S_4	D_1	D_2
	S_1	0	4	20	5	25	12
	S_2	10	0	6	10	5	20
	S_3	15	20	0	8	45	7
	S_4	20	25	10	0	30	6
	D_1	20	18	60	15	0	10
	D_2	10	25	30	23	4	0

Question 6

A list of activities along with their precedence requirement, normal time and cost and crash time and cost are given in the Table 6.

(i) Draw the network.

(ii) What are the normal and crash costs and durations of the project ? How many days will be saved and what will be the project cost if all the activities are crashed to the maximum possible extent ? It is expected that earlier completion of the project will result in additional profit of Rs. 50 per day. The indirect cost is Rs. 80 per day.

(iii) Analyse the project for optimum duration cost.

TABLE 6

Activity	Predecessor	Normal		Crash	
		Time (days)	Cost(Rs.)	Time (days)	Cost(Rs.)
A	—	4	300	2	450
B	A	9	600	5	960
C	A	6	620	4	780
D	B	4	320	3	395
E	B, D	6	1440	3	1,980
F	C, D	4	350	2	470

G	E, F	3	270	2	335

Question 7

(a)There are two competing departmental stores R and C in a city. Both stores have equal reputation and the total number of customers is equally divided between the two. Both the stores plan to run annual discount sales in the last week of December. For this, they want to attract more number of customers by using advertisement through newspaper, radio and television. By seeing the market trend, the store R constructed the following payoff matrix where the numbers in the matrix indicate a gain or a loss of customers.

	Store C		
Store R	40	50	-70
	10	25	-10
	100	30	60

(i)Check whether game is strictly determinable? If so find value of game.

(ii) Use Dominance property to reduce the given payoff matrix to 2x2.

(iii) Hence or otherwise, find optimal strategies for stores R and C.

(b) The vertices in the network shown in Fig.1 represent eight off shore oil wells. The edges represent possible connections that can be made between the wells. The weight on each edge is proportional to the estimated cost of constructing that link. We want to design a network that will connect the wells at minimum cost.

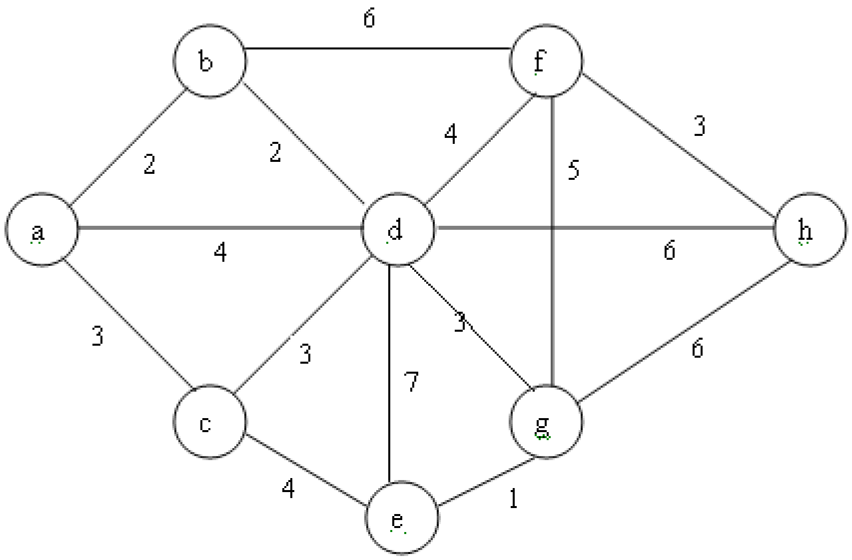


Fig. 1 Net work of eight off shore oil wells

Question 8

The captain of a cricket team has to allot five middle batting positions to five batsmen. The average runs scored by each batsman at these positions are as follows:

TABLE 7

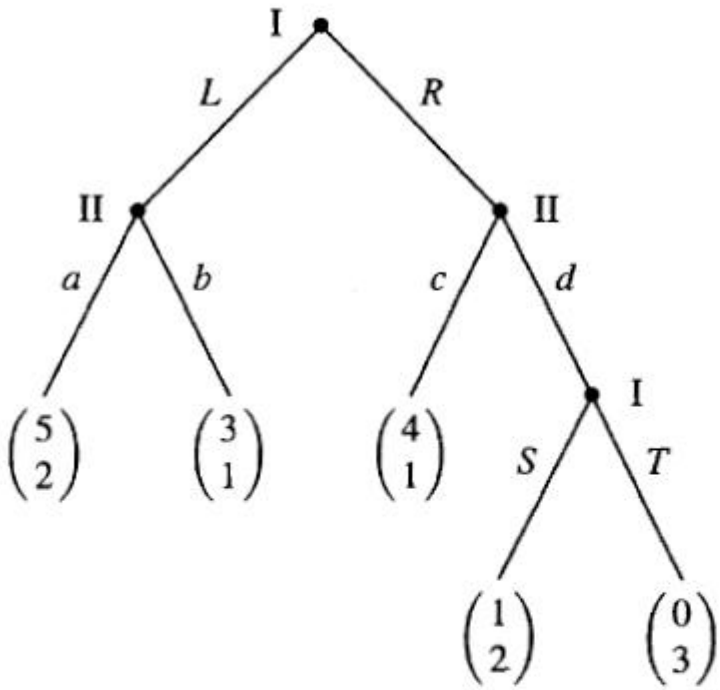
		Batting position				
		I	II	III	IV	V
Batsman	P	40	40	35	25	50
	Q	42	30	16	25	27
	R	50	48	40	60	50
	S	20	19	20	18	25
	T	58	60	59	55	53

(i) Find the assignment of batsmen to positions which would give the maximum number of runs.

(U) If another batsman 'U' with the following average runs in batting positions as given below:

Batting positions	I	11	III	IV	V
Average runs	45	52	38	50	49

is added to the team, should he be included to play in the team? If so, who will be replaced by him?



The flight timings between two cities, X and Y are as given in the following two tables. The minimum layover time of any crew in either of the cities is 3 hours. Determine the base city for each crew so that the sum of the layover times of all the crews in non-base cities is minimized.

Timings of Flights from City X to City Y

Flight number	Departure time (from City X)	Arrival time (to City Y)
101	6 a.m.	8.00 a. m.
102	10 a.m.	12.00 noon
103	3 p.m.	5.00 p.m.
104	8 p.m.	10.00 p.m.

Timings of Flights from City Y to City X

Flight number	Departure time (from City Y)	Arrival time (to City X)
201	5.30 a.m.	7.00 a.m.
202	9.00 a.m.	10.30 a.m.
203	4.00 p.m.	5.30 p.m.
204	10.00 p.m.	11.30 p.m.