OPERATIONAL RESEARCH

The first formal activities of operations research were initiate in England during world war II when a team of British scientists set out to make decisions regarding the best utilization of war materials. Following the end of the war, the ideas advanced in military operations were adapted to improve efficiency and productivity in the civilization sector. Today, OR is a dominant and indispensable decision-making tool.

Phases of an OR study

The principal phases for implementing OR in practice include

1. Definition of the problem
2. Construction of the problem
3. Solution of the model
4. validation of the model
5. Implementation of the solution

Models in operations research

A model in OR is a simplified representation of an operation, or is a process in which only the basic aspects or the most important features of a typical problem under investigation are considered.

In general, the first step in any model is the definition of the **alternatives** or the **decisions variables** of the problem. Next, decision variables are used to construct the **objective function** and the **constraints** of the model.

General format of the OR model

Maximize or Minimize **objective function**

Subject to

Constraints

A solution of the model is **feasible** if it satisfies all the constraints. It is **optimal** if, in addition to being feasible, it yields the best (minimum or maximum) value of the objective function.

Linear programming (LP)

Linear programming applies to optimization models in which the objective and constraints functions are strictly linear. Here we start with two-variable model. The **graphical method** is applicable to solve two-variable problems. The most widely used method for solving LP problems consisting of any number of variables is called **simplex method.**

Formulation of LP Problems

The procedure for mathematical formulation of a LPP consists of the following steps:

1. write down the decision variables of the problem
2. Formulate the objective function to be optimized as a linear function of the decision variables.
3. Formulate the other conditions of the problem such as resource limitations, market constraints etc, as linear in equations or in terms of decision variables.

4. Add non negative constraints from the considerations so that the negative values of the decision variables do not have any valid physical interpretation.

Eg 1: A manufacturer produces two types of models M_1 and M_2. Each model of the type M_1 requires 4 hrs of grinding and 2 hrs of polishing and M_2 requires 2 hrs of grinding and 5 hrs of polishing. The manufacturers have 2 grinders and 3 polishers. Each grinder works 40 hrs a week and each polisher works for 60 hrs a week. Profit on M_1 model is Rs. 3 and on model M_2 is Rs.4. Whatever is produced in a week is sold in the market. How should the manufacturer allocate his production capacity to the two types of models, so that he may make the maximum profit in a week?

Eg 2: A manufacturer produces both interior and exterior paints from two materials, A and B. The following table provides the basic data of the problem:

	Tons of raw material per ton of		maximum daily
	Exterior paint	interior paint	availability(tons)
Raw material A	6	4	24
Raw material B	1	2	6
Profit per ton(Rs. 1000)	5	4	

A market survey indicates that the daily demand for interior paint cannot exceed that of exterior paints by more than 1 ton. Also, the maximum daily demand of interior paint is 2 tons.

Manufacturer wants to determine the optimum product mix of interior and exterior paints that maximizes the total daily profit.

Graphical LP solution

The graphical procedure includes two steps:

1. Determination of the solution that defines all feasible solutions of the model.
2. Determination of the optimum solution from among all the feasible points in the solution space.

Eg 3:

A farmer uses at least 800 lb of special feed daily. The special feed is a mixture of corn and bean with the following compositions:

	lb Per lb of feedstuff		
Feedstuff	Protein	Fiber	Cost(Rs/lb)
Corn	0.09	0.02	0.30
Bean	0.6	0.06	0.90

The dietary requirements of the special feed are at least 30% protein and at most 5% fiber. The farmer wishes to determine the daily minimum cost feed mix.

Eg 4:

Maximize $z = 3x_1 + x_2$

Subject to
$$x_2 \le 5$$
$$x_1 + x_2 \le 10$$
$$-x_1 + x_2 \ge -2$$
$$x_1, x_2 \ge 0$$

Eg 5: Minimize $z = x_1 + x_2$

Subject to $3x_1 + x_2 \ge 6$
$$x_2 \ge 3$$
$$x_1 \le 4$$
$$x_1, x_2 \ge 0$$

Eg 6:

Maximize $z = x_1 + 2x_2$

Subject to
$$x_1 \le 6$$
$$x_1 + 2x_2 \le 8$$
$$-x_1 + x_2 \le 2$$
$$x_1, x_2 \ge 0$$

Eg 7: A person requires 10,12, and 12 units chemicals A,B and C respectively for his garden. One unit of liquid product contains 5,2 and 1 units of A,B, and C respectively. One unit of dry product contains 1,2 and 4 units of A,B,C. If the liquid product sells for Rs.3 and the dry product sells for rs.2, how many of each should be purchased, in order to minimize the cost and meet the requirements?

Eg 8: A paper mill produces two grades of paper namely A and B. Because of raw material restrictions, it cannot produce more than 400 tones of grade A and 300 tones of grade B in a week. There are 160 production hours in a week. It requires 0.2 and 0.4 hours to produce a ton of products A and B respectively with corresponding profits of Rs. 200 and Rs.500 per ton. Formulate the above as a LPP to maximize profit and find the optimum product mix.